

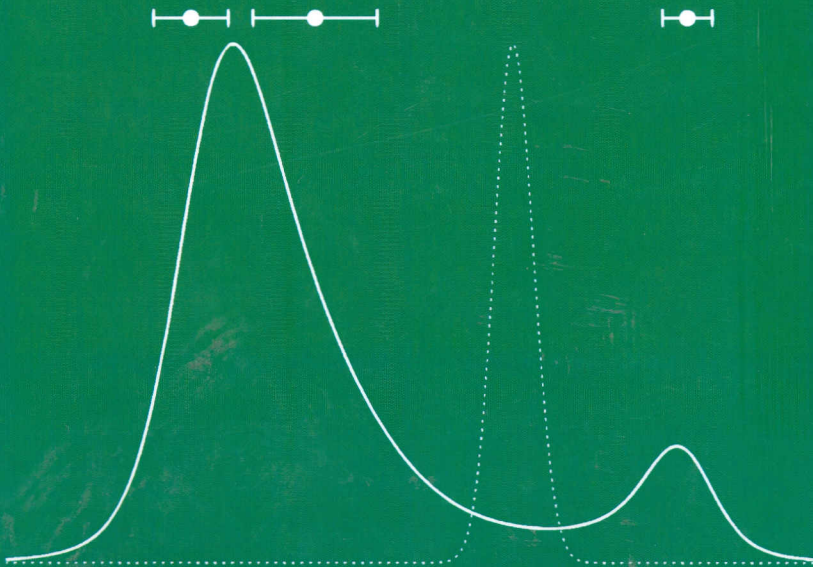
OXFORD SCIENCE PUBLICATIONS

# DATA ANALYSIS

## A BAYESIAN TUTORIAL

### SECOND EDITION

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with J. SKILLING



FOR SCIENTISTS AND  
ENGINEERS

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