

Rudolf Fritsch *Gerda Fritsch*

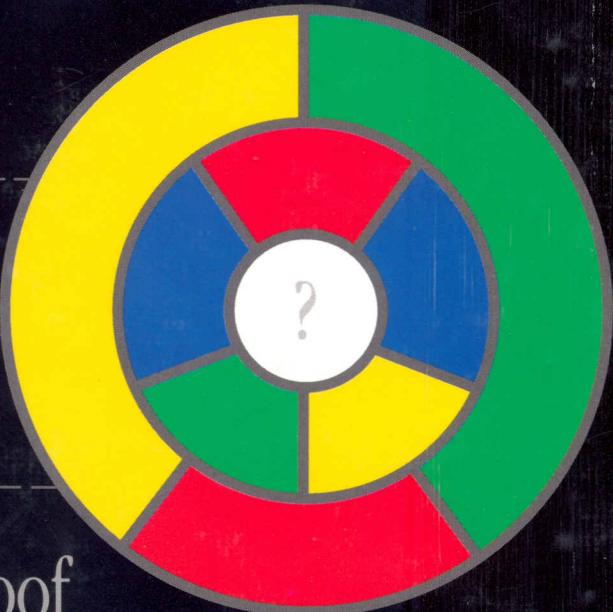
The Four-Color Theorem

History,

Topological

Foundations,

and Idea of Proof



Springer

Contents

Preface	vii
Acknowledgments for the English Edition	xi
Ways to Read This Book	xiii
1 History	1
2 (Topological) Maps	43
2.1 Preliminary Considerations	43
2.2 Borderlines	46
2.3 The Formal Definition of Maps	59
2.4 Basic Examples	68
2.5 National Borders	73
2.6 Common Borderlines	77
2.7 The Extension of Maps	81
3 The Four-Color Theorem (Topological Version)	85
3.1 Formulation and Basic Approach	85
3.2 First Steps Towards the Proof	87

4 Topology to Combinatorics	99
4.1 Complete (Plane) Graphs	99
4.2 The Wagner and Fáry Theorem	105
4.3 The Euler Polyhedral Formula	114
4.4 Duality	118
4.5 Cubic Maps	125
4.6 Counting Arguments	129
4.7 The Five-Color Theorem	133
4.8 Tait's Reformulation	134
5 The Four-Color Theorem (Combinatorial Version)	139
5.1 Vertex Colorings	139
5.2 Planar Graphs	141
5.3 Formulation and Further Advances	149
5.4 Rings and Configurations	152
5.5 And Now, to the Proof!	168
6 Reducibility	171
6.1 Kempe Chain Games	171
6.2 The Birkhoff Number and Some History	184
6.3 Types of Reducibility	186
6.4 The Dürre -Heesch Algorithm	187
6.5 A -, B -, and C -Reducibility	208
7 The Quest for Unavoidable Sets	219
7.1 Obstructions and a "Rule of Thumb"	219
7.2 Discharging Procedures	223
Bibliography	231
Works of Reference	249
Index	251