

# Colloidal Suspension Rheology

**Jan Mewis and  
Norman J. Wagner**

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The present book, which is intended as a general introduction to colloidal suspensions, is designed for a beginning in the field. Its purpose is to provide a systematic treatment of the established, central elements of the field. Practical examples are presented and discussed within a framework for understanding the underlying structure-property relationships. Emphasis is on understanding the various phenomena that contribute to the rheological properties of colloidal suspensions, such as available interactions and