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# Josephson Waveforms Characterization of a Sigma-Delta Analog-to-Digital Converter for Data Acquisition in Metrology

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