

# System Identification

## Theory for the User

Second Edition

Lennart Ljung  
Linköping University  
Sweden



**PRENTICE HALL PTR**  
Upper Saddle River, NJ 07458  
<http://www.phptr.com>

# Contents

<b>Preface to the First Edition</b> .....	<b>xiv</b>
<b>Acknowledgments</b> .....	<b>xvi</b>
<b>Preface to the Second Edition</b> .....	<b>xviii</b>
<b>Operators and Notational Conventions</b> .....	<b>xix</b>
<b>► 1 Introduction</b> .....	<b>1</b>
1.1 Dynamic Systems	1
1.2 Models	6
1.3 An Archetypical Problem—ARX Models and the Linear Least Squares Method	8
1.4 The System Identification Procedure	13
1.5 Organization of the Book	14
1.6 Bibliography	16
<b>part i: systems and models</b> .....	
<b>► 2 Time-Invariant Linear Systems</b> .....	<b>18</b>
2.1 Impulse Responses, Disturbances, and Transfer Functions	18
2.2 Frequency-Domain Expressions	28
2.3 Signal Spectra	33
2.4 Single Realization Behavior and Ergodicity Results (*)	42
2.5 Multivariable Systems (*)	44
2.6 Summary	45
2.7 Bibliography	46
2.8 Problems	47
Appendix 2A: Proof of Theorem 2.2	52
Appendix 2B: Proof of Theorem 2.3	55
Appendix 2C: Covariance Formulas	61

▶	3	Simulation and Prediction	63
3.1		Simulation	63
3.2		Prediction	64
3.3		Observers	72
3.4		Summary	75
3.5		Bibliography	75
3.6		Problems	76
▶	4	Models of Linear Time-Invariant Systems	79
4.1		Linear Models and Sets of Linear Models	79
4.2		A Family of Transfer-Function Models	81
4.3		State-Space Models	93
4.4		Distributed Parameter Models (*)	103
4.5		Model Sets, Model Structures, and Identifiability: Some Formal Aspects (*)	105
4.6		Identifiability of Some Model Structures	114
4.7		Summary	118
4.8		Bibliography	119
4.9		Problems	121
		Appendix 4A: Identifiability of Black-Box Multivariable Model Structures	128
▶	5	Models for Time-varying and Nonlinear Systems	140
5.1		Linear Time-Varying Models	140
5.2		Models with Nonlinearities	143
5.3		Nonlinear State-Space Models	146
5.4		Nonlinear Black-Box Models: Basic Principles	148
5.5		Nonlinear Black-Box Models: Neural Networks, Wavelets and Classical Models	154
5.6		Fuzzy Models	156
5.7		Formal Characterization of Models (*)	161
5.8		Summary	164
5.9		Bibliography	165
5.10		Problems	165

**part ii: methods**

---

▶	6	Nonparametric Time- and Frequency-Domain Methods	168
<hr/>			
6.1	Transient-Response Analysis and Correlation Analysis	168	
6.2	Frequency-Response Analysis	170	
6.3	Fourier Analysis	173	
6.4	Spectral Analysis	178	
6.5	Estimating the Disturbance Spectrum (*)	187	
6.6	Summary	189	
6.7	Bibliography	190	
6.8	Problems	191	
	Appendix 6A: Derivation of the Asymptotic Properties of the Spectral Analysis Estimate	194	
<hr/>			
▶	7	Parameter Estimation Methods	197
<hr/>			
7.1	Guiding Principles Behind Parameter Estimation Methods	197	
7.2	Minimizing Prediction Errors	199	
7.3	Linear Regressions and the Least-Squares Method	203	
7.4	A Statistical Framework for Parameter Estimation and the Maximum Likelihood Method	212	
7.5	Correlating Prediction Errors with Past Data	222	
7.6	Instrumental-Variable Methods	224	
7.7	Using Frequency Domain Data to Fit Linear Models (*)	227	
7.8	Summary	233	
7.9	Bibliography	234	
7.10	Problems	236	
	Appendix 7A: Proof of the Cramér-Rao Inequality	245	
<hr/>			
▶	8	Convergence and Consistency	247
<hr/>			
8.1	Introduction	247	
8.2	Conditions on the Data Set	249	
8.3	Prediction-Error Approach	253	
8.4	Consistency and Identifiability	258	
8.5	Linear Time-Invariant Models: A Frequency-Domain Description of the Limit Model	263	
8.6	The Correlation Approach	269	
8.7	Summary	273	
8.8	Bibliography	274	
8.9	Problems	275	

▶	9	Asymptotic Distribution of Parameter Estimates	280
<hr/>			
	9.1	Introduction	280
	9.2	The Prediction-Error Approach: Basic Theorem	281
	9.3	Expressions for the Asymptotic Variance	283
	9.4	Frequency-Domain Expressions for the Asymptotic Variance	290
	9.5	The Correlation Approach	296
	9.6	Use and Relevance of Asymptotic Variance Expressions	302
	9.7	Summary	304
	9.8	Bibliography	305
	9.9	Problems	305
		Appendix 9A: Proof of Theorem 9.1	309
		Appendix 9B: The Asymptotic Parameter Variance	313
<hr/>			
▶	10	Computing the Estimate	317
<hr/>			
	10.1	Linear Regressions and Least Squares	317
	10.2	Numerical Solution by Iterative Search Methods	326
	10.3	Computing Gradients	329
	10.4	Two-Stage and Multistage Methods	333
	10.5	Local Solutions and Initial Values	338
	10.6	Subspace Methods for Estimating State Space Models	340
	10.7	Summary	351
	10.8	Bibliography	352
	10.9	Problems	353
<hr/>			
▶	11	Recursive Estimation Methods	361
<hr/>			
	11.1	Introduction	361
	11.2	The Recursive Least-Squares Algorithm	363
	11.3	The Recursive IV Method	369
	11.4	Recursive Prediction-Error Methods	370
	11.5	Recursive Pseudolinear Regressions	374
	11.6	The Choice of Updating Step	376
	11.7	Implementation	382
	11.8	Summary	386
	11.9	Bibliography	387
	11.10	Problems	388
		Appendix 11A: Techniques for Asymptotic Analysis of Recursive Algorithms	389
		11A Problems	398

**part iii: user's choices**

---

▶	12 Options and Objectives	399
12.1	Options	399
12.2	Objectives	400
12.3	Bias and Variance	404
12.4	Summary	406
12.5	Bibliography	406
12.6	Problems	406
▶	13 Experiment Design	408
13.1	Some General Considerations	408
13.2	Informative Experiments	411
13.3	Input Design for Open Loop Experiments	415
13.4	Identification in Closed Loop: Identifiability	428
13.5	Approaches to Closed Loop Identification	434
13.6	Optimal Experiment Design for High-Order Black-Box Models	441
13.7	Choice of Sampling Interval and Presampling Filters	444
13.8	Summary	452
13.9	Bibliography	453
13.10	Problems	454
▶	14 Preprocessing Data	458
14.1	Drifts and Detrending	458
14.2	Outliers and Missing Data	461
14.3	Selecting Segments of Data and Merging Experiments	464
14.4	Prefiltering	466
14.5	Formal Design of Prefiltering and Input Properties	470
14.6	Summary	474
14.7	Bibliography	475
14.8	Problems	475
▶	15 Choice of Identification Criterion	477
15.1	General Aspects	477
15.2	Choice of Norm: Robustness	479
15.3	Variance-Optimal Instruments	485
15.4	Summary	488

15.5	Bibliography	489
15.6	Problems	490
▶	<b>16 Model Structure Selection and Model Validation</b>	<b>491</b>
16.1	General Aspects of the Choice of Model Structure	491
16.2	A Priori Considerations	493
16.3	Model Structure Selection Based on Preliminary Data Analysis	495
16.4	Comparing Model Structures	498
16.5	Model Validation	509
16.6	Residual Analysis	511
16.7	Summary	516
16.8	Bibliography	517
16.9	Problems	518
▶	<b>17 System Identification in Practice</b>	<b>520</b>
17.1	The Tool: Interactive Software	520
17.2	The Practical Side of System Identification	522
17.3	Some Applications	525
17.4	What Does System Identification Have To Offer?	536
▶	<b>Appendix I Some Concepts From Probability Theory</b>	<b>539</b>
▶	<b>Appendix II Some Statistical Techniques for Linear Regressions</b>	<b>543</b>
II.1	Linear Regressions and the Least Squares Estimate	543
II.2	Statistical Properties of the Least-Squares Estimate	551
II.3	Some Further Topics in Least-Squares Estimation	559
II.4	Problems	564
	<b>References</b>	<b>565</b>
	<b>Subject Index</b>	<b>596</b>
	<b>Reference Index</b>	<b>603</b>